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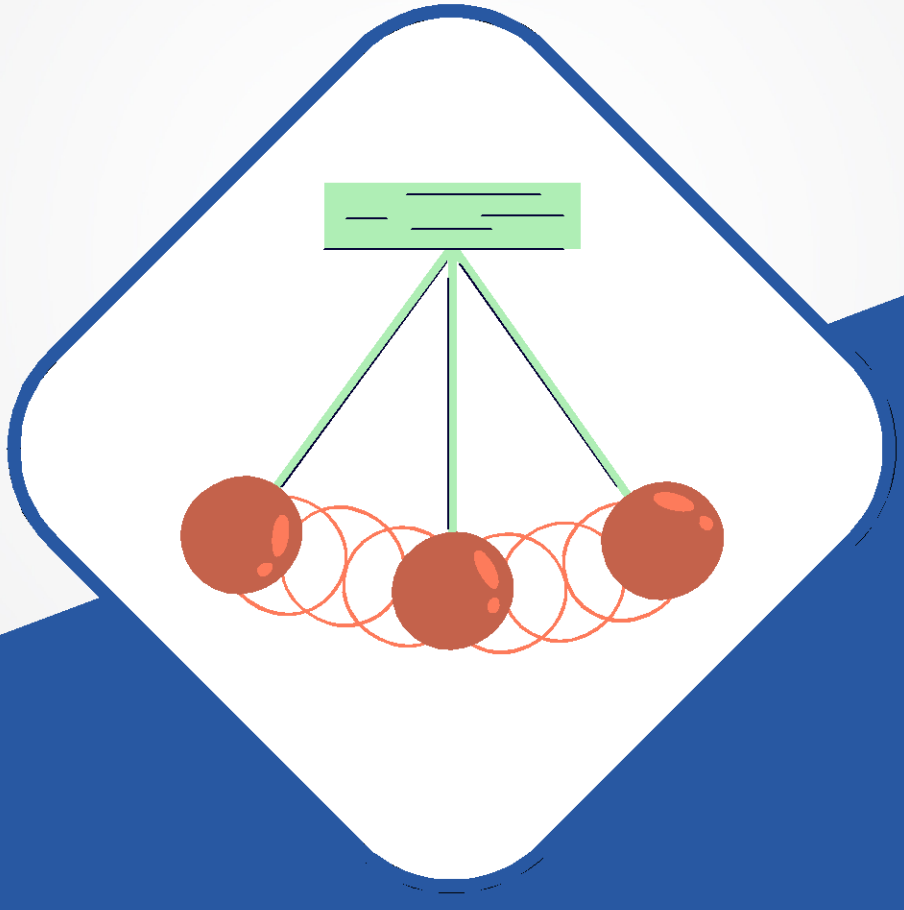
ENTHUSIAST COURSE



STUDY MATERIAL

ELECTROSTATICS

English Medium

ALLEN®

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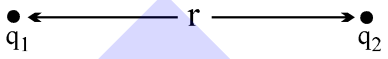
ELECTROSTATICS

ELECTRIC CHARGE

Charge of a material body is that property due to which it interacts with other charges. There are two kinds of charges- positive and negative. S.I. unit is coulomb. Charge is quantized, conserved, and additive.

COULOMB'S LAW

$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}$. In vector form $\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^3}\vec{r}$



where ε_0 = permittivity of free space = 8.85×10^{-12} N^{-1} m^{-2} c^2 or F/m

NOTE :

The Law is applicable only for static and point charges.

Moving charges may result in magnetic interaction. And if charges are extended, induction may change the charge distribution.

❖ **Principle Of Superposition**

Force on a point charge due to many charges is given by $\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + ...$

NOTE : The force due to one charge is not affected by the presence of other charges.

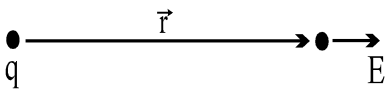
❖ **Electric Field, Electric Intensity Or Electric Field Strength (Vector Quantity)**

In the surrounding region of a charge there exist a physical property due to which other charge experiences a force. The direction of electric field is direction of force experienced by a positively charged particle and the magnitude of the field (electric field intensity) is the force experienced by a unit charge.

$\vec{E} = \frac{\vec{F}}{q}$ unit is N/C or V/m.

❖ **Electric Field Due To**

(i) Point charge : $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r} = \frac{1}{4\pi\in_0}\frac{q}{r^3}\vec{r}$ (vector form)



Where $\vec{r}$ = vector drawn from the source charge to the point

value of q must be substituted with sign.

(ii) Continuous charge distribution $\vec{E} = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r^2}\hat{r} = \int d\vec{E}$

$d\vec{E}$ = electric field due to an elementary charge.

Note : $E \neq \int dE$ because E is a vector quantity.

$dq = \lambda d\ell$ (for line charge) = σds (for surface charge) = ρdv (for volume charge)

In general λ, σ & ρ are linear, surface and volume charge densities respectively.

(iii) Infinite line of charge $|\vec{E}| = \frac{2k\lambda}{r}$ where r = perpendicular distance of the point from the line charge.

(iv) Semi ∞ line of charge $|\vec{E}| = \frac{\sqrt{2}k\lambda}{r}$ as, $E_x = \frac{k\lambda}{r}$ & $E_y = \frac{k\lambda}{r}$ at a point above the end of wire at an angle 45°.

(v) Uniformly charged ring, $E_{centre} = 0$, $E_{axis} = \frac{K\,Q\,x}{(x^2 + R^2)^{3/2}}$

(vi) Electric field is maximum when $\frac{dE}{dx} = 0$ for a point on the axis of the ring. Here we get $x = R/\sqrt{2}$.

(vii) Infinite non conducting sheet of charge $\vec{E} = \frac{\sigma}{2\varepsilon_0}\hat{n}$ where $\hat{n}$ = unit vector normal to the plane of sheet, where σ is surface charge density.

(viii) Infinite charged conducting sheet having surface charge density σ on each of surfaces $E = \sigma/\varepsilon_o$.

(ix) Just outside a conducting surface charged with a surface charge density σ, electric field is always given as $E = \sigma/\varepsilon_o$.

(x) Uniformly charged solid sphere (Insulating material)

$E_{out} = \frac{Q}{4\pi\varepsilon_0 r^2}$; r $\geq$ R , Behaves as a point charge situated at the centre for these points

$E_{in} = \frac{Qr}{4\pi\varepsilon_0 R^3} = \frac{\rho r}{3\varepsilon_0}$; $r \leq R$ where ρ = volume charge density

(xi) Uniformly charged spherical shell (conducting) or uniformly charged solid conducting sphere.

$E_{out} = \frac{Q}{4\pi\varepsilon_0 r^2}$; $r \geq R$ Behaves as a point charge situated at the centre for these points.

$E_{in} = 0$; $r < R$

(xii) Uniformly charged cylinder with a charge density ρ is $E_{in} = \frac{\rho r}{2\,\epsilon_0}$; for $r < R$

$E_{out} = \frac{\rho R^2}{2\,\epsilon_0\, r}$; for $r > R$

(xiii) Uniformly charged cylindrical shell with surface charge density σ is

$E_{in} = 0$; for $r < R$ $E_{out} = \frac{\rho r}{\epsilon_0\, r}$; for $r > R$

ELECTRIC FIELD LINES (EFL)

Electric field lines are pictorial representation of electric field. They are drawn in such a way that tangent at any point is in the direction of electric field at that point. Therefore an arrow is put on electric field line pointing away from positive charge and towards the negative charge.

❖ **Properties of (EFL) :**

(i) Electric field lines never intersects but they can touch each other.

(ii) EFL originates from positive charge and terminate on a negative charge. If an EFL is originated, it must require termination either at a negative charge or at ∞ .

(iii) In a charge free region electric field lines are continuous.

(iv) Number of EFL originated or terminated from a charge or on a charge is proportional to the magnitude of charge.

❖ **Electric Potential** ***(Scalar Quantity)***

Work done by external agent to bring a unit positive charge (without acceleration) from infinity to a point in an electric field is called electric potential at that point . If $W_{\infty r}$ is the work done to bring a charge q (very small) from infinity to a point then potential at that point is $V = \frac{(W_{\infty r})_{ext}}{q}$; S.I. unit is volt (= 1 J/C)

❖ **Potential Difference**

$V_{AB} = V_A - V_B = \frac{(W_{BA})_{ext}}{q}$ V_{AB} = potential difference between point A & B.

W_{BA}= work done by external source to transfer a point charge q from B to A (Without acceleration).

❖ **Electric Field & Electric Potential**

$\vec{E} = -\text{grad}\, V = -\vec{\nabla} V$ {read as gradient of V} grad $= \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$

Here electric potential varies with x, y and z coordinates.

For finding potential difference between two points in electric field, we use

$V_A - V_B = -\int_A^B \vec{E}.\vec{d\ell}$ if E is varying with distance

$= -Ed$ if E is constant & here d is the distance between points A and B.

❖ **Potential Due To**

(i) A point charge $V = \frac{Q}{4\pi\varepsilon_0 r}$

(ii) Many charges $V = \frac{q_1}{4\pi\varepsilon_0 r_1} + \frac{q_2}{4\pi\varepsilon_0 r_2} + \frac{q_3}{4\pi\varepsilon_0 r_3} + ...$

(iii) Continuous charge distribution $V = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r}$

(iv) Spherical shell (conducting or non conducting) or solid conducting sphere

$$V_{out} = \frac{Q}{4\pi\varepsilon_0 r} \; ; \; (r \geq R) \quad , \quad V_{in} = \frac{Q}{4\pi\varepsilon_0 R} \; ; \; (r \leq R)$$

(v) Non conducting uniformly charged solid sphere :

$$V_{out} = \frac{Q}{4\pi\varepsilon_0 r} \; ; \; (r \geq R) \, , \quad V_{in} = \frac{1}{2}\frac{Q(3R^2 - r^2)}{4\pi\varepsilon_0 R^3} \; ; \; (r \leq R)$$

❖ **Equipotential Surface And Equipotential Region**

In an electric field the locus of points of equal potential is called an equipotential surface. An equipotential surface and the electric field meet at right angles. The region where $E = 0$, Potential of the whole region must remain constant as no work is done in displacement of charge in it. It is called as equipotential region like conducting bodies.

❖ **Mutual Potential Energy Or Interaction Energy**

"The work to be done to integrate the charge system".

For 2 particle system $U_{\text{mutual}} = \frac{q_1 q_2}{4\pi\varepsilon_0 r}$

For 3 particle system $U_{\text{mutual}} = \frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}} + \frac{q_2 q_3}{4\pi\varepsilon_0 r_{23}} + \frac{q_3 q_1}{4\pi\varepsilon_0 r_{31}}$

For n particles there will be $\frac{n\,(n-1)}{2}$ terms .

Total energy of a system = $U_{\text{self}} + U_{\text{mutual}}$

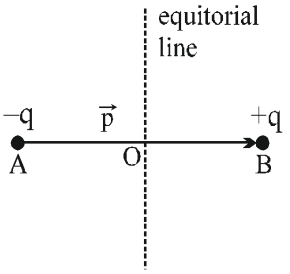
❖ Potential energy of charge q in potential field $U = qV$

Interaction energy of a system of two charges $U = q_1 V_2 = q_2 V_1$

ELECTRIC DIPOLE

(a) Dipole moment $\vec{p} = q\vec{d}$

(d is the separation between the charges and from $-q$ to $+q$)



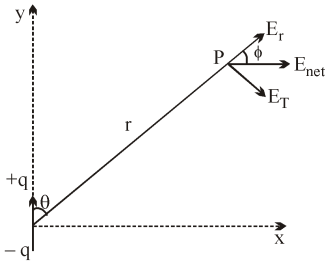
(b) Electric field at a general point $P(r, \theta)$ in polar co-ordinate system is

Radial electric field $E_r = \frac{2Kp\cos\theta}{r^3}$

Tangential electric field $E_T = \frac{Kp\sin\theta}{r^3}$

Net electric field at P is $E_{net} = \sqrt{E_r^2 + E_T^2} = \frac{kp}{r^3}\sqrt{1+3\cos^2\theta}$

$$\tan\phi = \frac{1}{2}\tan\theta$$



(c) Electric Potential at point P is $V_P = \frac{Kp\cos\theta}{r^2} = \frac{\vec{p}\cdot\vec{r}}{4\pi\varepsilon_0 r^3}$

(d) Electric Dipole in uniform electric field : Torque $\vec{\tau} = \vec{p} \times \vec{E}$; $\vec{F} = 0$.

(e) Work done in rotation of dipole is $W = PE(\cos\theta_1 - \cos\theta_2)$

(f) Potential energy of an electric dipole in electric field $U = -\vec{p}\cdot\vec{E}$.

ELECTRIC FLUX

(i) For uniform electric field; $\phi = \vec{E}.\vec{A} = EA\cos\theta$ where θ = angle between $\vec{E}$ & area vector $(\vec{A})$. Flux is contributed only due to the component of electric field which is perpendicular to the plane.

(ii) If $\vec{E}$ is not uniform throughout the area A, then $\phi = \int\vec{E}.d\vec{A}$

GAUSS'S LAW

(Applicable only to closed surface) Net flux emerging out of a closed surface is $\frac{q_{en}}{\varepsilon_0}$.

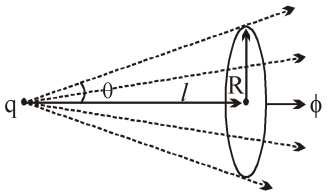
$$\phi = \oint\vec{E}\cdot d\vec{A} = \frac{q_{en}}{\varepsilon_0}$$

q = net charge enclosed by the closed surface .

ϕ does not depend on the

(i) Shape and size of the closed surface

(ii) The charges located outside the closed surface.



Solid angle of cone of half angle θ is $\Omega = 2\pi(1-\cos\theta)$

CONCEPT OF SOLID ANGLE :

Flux of charge q having through the circle of radius R is $\phi = \frac{q/\epsilon_0}{4\pi}\times\Omega = \frac{q}{2\epsilon_0}(1-\cos\theta)$

- In vacuum energy stored per unit volume in an electric field $= \frac{\varepsilon_o E^2}{2}$
- Electric pressure due to its own charge on a surface having charged density σ is $P_{ele} = \frac{\sigma^2}{2\varepsilon_o}$.
- Electric pressure on a charged surface with charged density σ due to external electric field is $P_{ele} = \sigma E_1$

❖ **Important Points To Be Remembered**

(i) Electric field is always perpendicular to a conducting surface (or any equipotential surface).

No tangential component of electric field on such surfaces.

(ii) When a conductor is charged, the charge resides only on the surface.

(iii) Charge density at convex sharp points on a conductor is greater. Lesser is radius of curvature at a convex part, greater is the charge density.

(iv) For a conductor of any shape E (just outside) = $\frac{\sigma}{\varepsilon_o}$

(v) Potential difference between two points in an electric field does not depend on the path between them.

(vi) Potential at a point due to positive charge is positive & due to negative charge is negative.

(vii) Positive charge flows from higher to lower (i.e. in the direction of electric field) potential and negative charge from lower to higher (i.e. opposite to the electric field) potential .

(viii) When $\vec{p} \parallel \vec{E}$ the dipole is in stable equilibrium

(ix) When $\vec{p} \parallel (-\vec{E})$ the dipole is in unstable equilibrium

(x) When a charged isolated conducting sphere is connected to an uncharged small conducting sphere then potential (and charge) remains almost same on the larger sphere while smaller is charged .

(xi) Self potential energy of a charged conducting spherical shell = $\frac{KQ^2}{2R}$.

(xii) Self potential energy of an insulating uniformly charged sphere = $\frac{3KQ^2}{5R}$.

(xiii) A spherically symmetric charge {i.e ρ depends only on r} behaves as if its charge is concentrated at its centre (for outside points).

(xiv) **Dielectric strength of material :** The minimum electric field required to ionize the medium or the maximum electric field which the medium can bear without breaking down.

SOLVED EXAMPLES

1. When a piece of polythene is rubbed with wool, a charge of -2×10^{-7} C is developed on polythene. What is the amount of mass, which is transferred to polythene.

Sol. From Q = ne, So, the number of electrons transferred $n = \frac{Q}{e} = \frac{2\times10^{-7}}{1.6\times10^{-19}} = 1.25 \times 10^{12}$

Now mass of transferred electrons = n mass of one electron

$= 1.25 \times 10^{12} \times 9.1 \times 10^{-31} = 11.38 \times 10^{-19}$ kg

2. If the distance between two equal point charges is doubled and their individual charges are also doubled, what would happen to the force between them?

Sol. $F = \frac{1}{4\pi\varepsilon_0}\frac{q\times q}{r^2}$ ....(i) Again, $F' = \frac{1}{4\pi\varepsilon_0}\frac{(2q)(2q)}{(2r)^2}$

or $F' = \frac{1}{4\pi\varepsilon_0}\frac{4q^2}{4r^2} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2} = F$

So, the force will remain the same.

3. A particle of mass m carrying charge '$+q_1$' is revolving around a fixed charge '$-q_2$' in a circular path of radius r. Calculate the period of revolution.

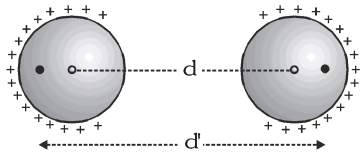
Sol. $\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2} = mr\omega^2 = \frac{4\pi^2mr}{T^2}$

$T^2 = \frac{(4\pi\varepsilon_0)r^2(4\pi^2mr)}{q_1q_2}$ or $T = 4\pi r\sqrt{\frac{\pi\varepsilon_0 mr}{q_1q_2}}$

where $\vec{r}$ is the vector drawn from source charge is test charge.

4. The force of repulsion between two point charges is F, when these are at a distance of 1 m. Now the point charges are replaced by spheres of radii 25 cm having the charge same as that of point charges. The distance between their centres is 1 m, then compare the force of repulsion in two cases.

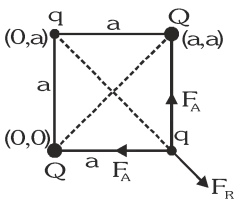
Sol. In 2nd case due to mutual repulsion, the effective distance between their centre of charges will be increased (d' > d) so force of repulsion decreases as $F \propto \frac{1}{d^2}$



5. For the system shown in figure find Q for which resultant force on q is zero.

Sol. For force on q to be zero, charges q and Q must be of opposite of nature.

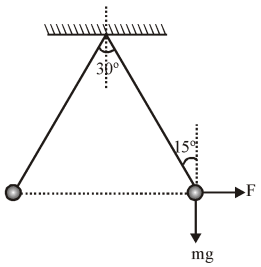
Net attraction force on q due to charges Q = Repulsion force on q due to q



$\sqrt{2}\ F_A = F_R \Rightarrow \sqrt{2}\ \frac{kQq}{a^2} = \frac{kq^2}{(\sqrt{2}a)^2} \Rightarrow q = 2\sqrt{2}\ Q$

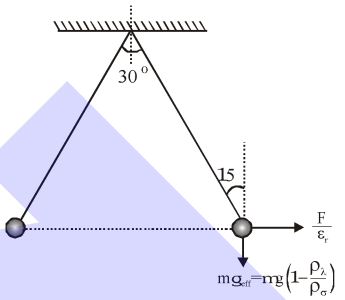
Hence $q = -2\sqrt{2}\,Q$

6. Two identically charged spheres are suspended by strings of equal length. The strings make an angle of 30^0 with each other. When suspended in a liquid of density 0.8 g/cc the angle remains same. What is the dielectric constant of liquid. Density of sphere = 1.6 g/cc.



Sol. When set up shown in figure is in air, we have $\tan 15^0 = \frac{F}{mg}$

When set up is immersed in the medium as shown in figure, the electric force experienced by the ball will reduce and will be equal to $\frac{F}{\varepsilon_r}$ and the effective gravitational force will become $mg\left(1-\frac{\rho_\ell}{\rho_s}\right)$ Thus we have $\tan 15^0 = \frac{F}{mg \in_r \left(1-\frac{\rho_\ell}{\rho_s}\right)} = \frac{F}{mg} \Rightarrow \in_r = \frac{1}{1-\frac{\rho_\ell}{\rho_s}} = 2$



7. Calculate the electric field intensity E which would be just sufficient to balance the weight of an electron. If this electric field is produced by a second electron located below the first one what would be the distance between them? [Given : e = 1.6×10^{-19} C, m = 9.1×10^{-31} kg and g = 9.8 m/s^2]

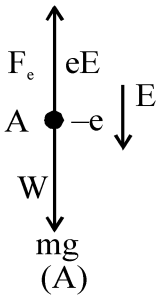
Sol. As force on a charge e in an electric field E

$F_e = eE$

So according to given problem

$F_e = W \Rightarrow eE = mg$

$\Rightarrow E = \frac{mg}{e} = \frac{9.1\times10^{-31}\times9.8}{1.6\times10^{-19}} = 5.57 \times 10^{-11} \frac{V}{m}$

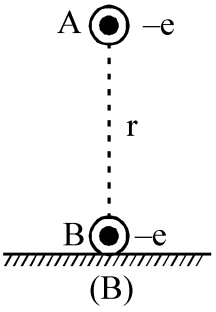


As this intensity E is produced by another electron B, located at a distance r below A.

$E = \frac{1}{4\pi\varepsilon_0}\frac{e}{r^2} \Rightarrow r = \sqrt{\frac{e}{4\pi\varepsilon_0 E}}$

So,

$r = \left[\frac{9\times10^9\times1.6\times10^{-19}}{5.57\times10^{-11}}\right]^{1/2} \simeq 5 \text{ m}$



8. If a point charge q is placed at the centre of a cube.

What is the flux linked (a) with the cube? (b) with each face of the cube?

Sol. (a) According to Gauss's law flux linked with a enclosed surface is $(1/\varepsilon_0)$ times the charge enclosed and here the enclosed surface cube is enclosing a charge q so, $\phi_T = \frac{1}{\varepsilon_0}(q)$

(b) Now as cube is a symmetrical surface with 6–faces and the point charge is at its centre, so electric flux linked with each face will be $\phi_F = \frac{1}{6}(\phi_T) = \frac{q}{6\varepsilon_0}$

Note: (i) Here flux linked with cube or one of its faces is independent of the side of cube.

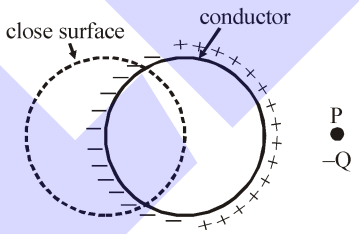
(ii) If charge is not at the centre of cube (but anywhere inside it), total flux will not change, but the flux linked with different faces will be different.

9. If a point charge q is placed at one corner of a cube, what is the flux linked with the cube?

Sol. In this case by placing three cubes at three sides of given cube and four cubes above, the charge will be in the centre. So, the flux linked with each cube will be one–eight of the flux $\frac{q}{\varepsilon_0}$.

$\therefore$ Flux associated with given cube = $\frac{q}{8\varepsilon_0}$

10. As shown in figure a closed surface intersects a spherical conductor. If a negative charge is placed at point P. What is the nature of the electric flux coming out of the closed surface ?



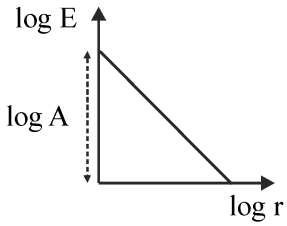
Sol. Point charge Q induces charge on conductor as shown in figure.

Net charge enclosed by closed surface is negative so flux is negative.

11. For infinite line distribution of charge draw the curve between log E and log r.

Sol. $\because E = \frac{\lambda}{2\pi \in_0 r} = \frac{A}{r}$ where $A = \frac{\lambda}{2\pi \in_0} = \text{constant}$

take log on both side log E = log A – log r



12. A charge 2μC is taken from infinity to a point in an electric field, without changing its velocity, if work done against electrostatic forces is –40μJ then potential at that point is ?

Sol. $V = \frac{W_{ext}}{q} = \frac{-40\mu J}{2\mu C} = -20V$

Note : Always remember to put sign of W and q.

13. 1μC charge is shifted from A to B and it is found that work done by external force is 80μJ against electrostic forces, find $V_A - V_B$

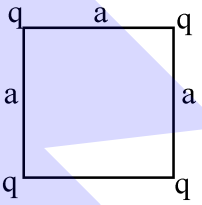
Sol. $W_{AB} = q(V_B - V_A)$, $80\ \mu J = 1\mu C\ (V_B - V_A) \Rightarrow V_A - V_B = -80\ V$

14. If $V = -5x + 3y + \sqrt{15}\,z$ then find magnitude of electric field at point (x,y,z).

Sol. $\vec{E} = -\left[\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right] = -(-5\,\hat{i} + 3\,\hat{j} + \sqrt{15}\,\hat{k}) \Rightarrow |\vec{E}| = \sqrt{25+9+15} = \sqrt{49} = 7\,\text{unit}$

15. The four charges q each are placed at the corners of a square of side a. Find the potential energy of one of the charges The electric potential of a point A due to charges B, C and D is

Sol. $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{a} + \frac{1}{4\pi\varepsilon_0}\frac{q}{\sqrt{2}a} + \frac{1}{4\pi\varepsilon_0}\frac{q}{a} = \frac{1}{4\pi\varepsilon_0}\left(2+\frac{1}{\sqrt{2}}\right)\frac{q}{a}$



$\therefore$ Potential energy of the charge at A is $PE = qV = \frac{1}{4\pi\varepsilon_0}\left(2+\frac{1}{\sqrt{2}}\right)\frac{q^2}{a}$.

16. A proton moves with a speed of 7.45×10^5 m/s directly towards a free proton originally at rest. Find the distance of closest approach for the two protons.

Given : $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\ \frac{N-m^2}{C^2}$; $m_p = 1.67 \times 10^{-27}$ kg and $e = 1.6 \times 10^{-19}$ C

Sol. As here the particle at rest is free to move, when one particle approaches the other, due to electrostatic repulsion other will also start moving and so the velocity of first particle will decrease while of other will increase and at closect approach both will move with same velocity. So if v is the common velocity of each particle at closest approach, by 'conservation of momentum'.

$mu = mv + mv \Rightarrow v = \frac{1}{2}u$

And by conservation of energy $\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + \frac{1}{2} + \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}$

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18. A system has two charges $q_A = 2.5 \times 10^{-7}$ C and $q_B = -2.5 \times 10^{-7}$ C located at points A: (0, 0, – 0.15 m) and B ; (0, 0, + 0.15 m) respectively. What is the total charge and electric dipole moment of the system?

Sol. Total charge = $2.5 \times 10^{-7} - 2.5 \times 10^{-7} = 0$

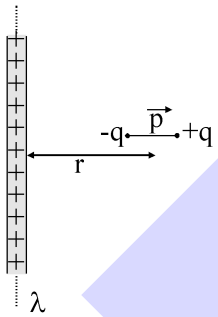
Electric diople moment,

p = Mangitude of either charge × separation between charges

$= 2.5 \times 10^{-7}$ [0.15 + 0.15] C m $= 7.5 \times 10^{-8}$ C m.

The direction of dipole moment is from B to A.

19. Calculate force on a dipole in the surrounding of a long charged wire as shown in the figure.



Sol. In the situation shown in figure, the electric field strength due to the wire, at the position of dipole as $E = \frac{2k\lambda}{r}$

Thus force on dipole is $F = -p.\frac{dE}{dr} = -p\left[-\frac{2k\lambda}{r^2}\right] = \frac{2kp\lambda}{r^2}$

Here –ve charge of dipole is close to wire hence net force an dipole due to wire will be attractive.

20. A short electric dipole is situated at the origin of coordinate axis with its axis along x–axis and equator along y–axis. It is found that the magnitudes of the electric intensity and electric potential due to the dipole are equal at a point distant r = $\sqrt{5}$ m from origin. Find the position vector of the point in first quadrant.

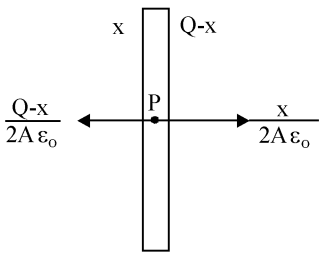
Sol. $\because |E_P| = |V_P| \therefore \frac{kp}{r^3}\sqrt{1+3\cos^2\theta} = \frac{kp\cos\theta}{r^2} \Rightarrow 1+3\cos^2\theta = 5\cos^2\theta \Rightarrow \cos\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = 45^0$

Position vector $\vec{r}$ of point P is $\vec{r} = \frac{\sqrt{5}}{2}\left(\tilde{i}+\tilde{j}\right)$

21. Prove that if an isolated (isolated means no charges are near the sheet) large conducting sheet is given a charge then the charge distributes equally on its two surfaces.

Sol. Let there is x charge on left side of sheet and Q–x charge on right side of sheet.

Since point P lies inside the conductor so $E_P = O$



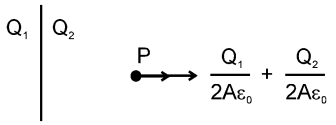
$$\frac{x}{2A\varepsilon_o} - \frac{Q-x}{2A\varepsilon_o} = 0 \Rightarrow \frac{2x}{2A\varepsilon_o} = \frac{Q}{2A\varepsilon_o} \Rightarrow x = \frac{Q}{2}$$

$$Q - x = \frac{Q}{2}$$

So charge is equally distributed on both sides

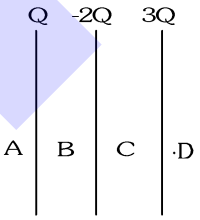
22. If an isolated infinite sheet contains charge Q_1 on its one surface and charge Q_2 on its other surface then prove that electric field intensity at a point in front of sheet will be $\frac{Q}{2A\varepsilon_0}$, where $Q = Q_1 + Q_2$

Sol. Electric field at point P : $\vec{E} = \vec{E}_{Q_1} + \vec{E}_{Q_2} = \frac{Q_1}{2A\varepsilon_0}\hat{n} + \frac{Q_2}{2A\varepsilon_0}\hat{n} = \frac{Q_1+Q_2}{2A\varepsilon_0}\hat{n} = \frac{Q}{2A\varepsilon_0}\hat{n}$



[This shows that the resultant field due to a sheet depends only on the total charge of the sheet and not on the distribution of charge on individual surfaces].

23. Three large conducting sheets placed parallel to each other at finite distance contains charges Q, –2Q and 3Q respectively. Find electric field at points A, B, C, and D



Sol. $E_A = E_Q + E_{-2Q} + E_{3Q}$.

(i) Here E_Q means electric field due to 'Q'.

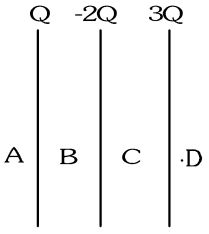
$E_A = \frac{(Q-2Q+3Q)}{2A\varepsilon_0} = \frac{2Q}{2A\varepsilon_0} = \frac{Q}{A\varepsilon_0}$, towards left

(ii) $E_B = \frac{Q-(-2Q+3Q)}{2A\varepsilon_0}$, towards right = 0

(iii) $E_C = \frac{(Q-2Q)-(3Q)}{2A\varepsilon_0} = \frac{-4Q}{2A\varepsilon_0} = \frac{-2Q}{A\varepsilon_0}$, towards right $\Rightarrow \frac{2Q}{A\varepsilon_0}$ towards left

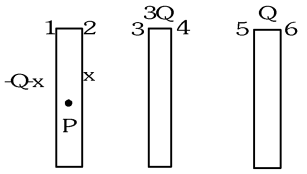
(iv) $E_D = \frac{(Q-2Q+3Q)}{2A\varepsilon_0} = \frac{2Q}{2A\varepsilon_0} = \frac{Q}{A\varepsilon_0}$, towards right

24. Figure shows three large metallic plates with charges –Q, 3Q and Q respectively. Determine the final charges on all the surfaces.



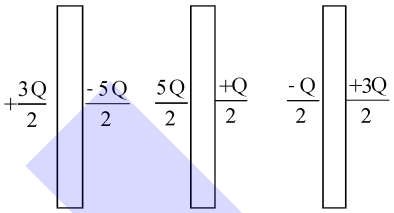
Sol. We assume that charge on surface 2 is x. Following conservation of charge, we see that surfaces 1 has charge $(-Q-x)$. The electric field inside the metal plate is zero so fields at P is zero.

Resultant field at P : $E_P = 0 \Rightarrow \frac{-Q-x}{2A\varepsilon_0}$



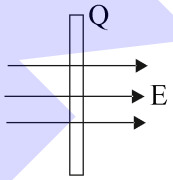
$= \frac{x+3Q+Q}{2A\varepsilon_0} \Rightarrow -Q-x = x+4Q \Rightarrow x = \frac{-5Q}{2}$

Note : We see that charges on the facing surfaces of the plates are of equal magnitude and opposite sign. This can be in general proved by gauss theorem also. Remember this it is important result.



Thus the final charge distribution on all the surfaces is :

25. An isolated conducting sheet of area A and carrying a charge Q is placed in a uniform electric field E, such that electric field is perpendicular to sheet and covers all the sheet. Find out charges appearing on its two surfaces. Also

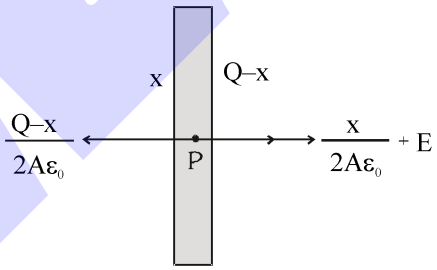


Sol. Let there is x charge on left side of plate and

Q – x charge on right side of plate

$E_P = 0 \Rightarrow \frac{x}{2A\varepsilon_0} + E = \frac{Q-x}{2A\varepsilon_0} \Rightarrow \frac{x}{A\varepsilon_0} = \frac{Q}{2A\varepsilon_0} - E$

$\Rightarrow x = \frac{Q}{2} - EA\varepsilon_0$

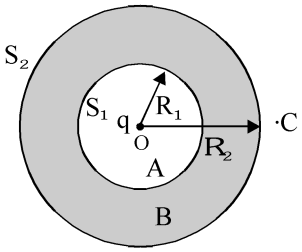


So charge on one side is $\frac{Q}{2} - EA\varepsilon_0$ and other side $\frac{Q}{2} + EA\varepsilon_0$

The resultant electric field on the left and right side of the plate.

On right side E = $\frac{Q}{2A\varepsilon_0}$ + E towards right and on left side $\frac{Q}{2A\varepsilon_0}$ – E towards left.

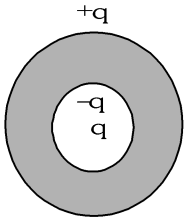
26. An uncharged conductor of inner radius R_1 and outer radius R_2 contains a point charge q at the centre as shown in figure



(i) Find $\vec{E}$ and V at points A,B and C

(ii) If a point charge Q is kept out side the sphere at a distance 'r' (>>R_2) from centre then find out resultant force on charge Q and charge q.

Sol. At point A : $V_A = \frac{Kq}{OA} + \frac{Kq}{R_2} + \frac{K(-q)}{R_1}$, $\vec{E}_A = \frac{Kq}{OA^3}\overrightarrow{OA}$



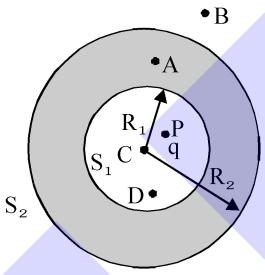
At point B : $V_B = \frac{Kq}{OB} + \frac{K(-q)}{OB} + \frac{Kq}{R_2} = \frac{Kq}{R_2}$, $E_B = 0$;

At point C : $V_C = \frac{Kq}{OC}$, $\vec{E}_C = \frac{Kq}{OC^3}\overrightarrow{OC}$

(ii) Force on point charge Q : $\vec{F}_Q = \frac{KqQ}{r^2}\hat{r}$ (r = distance of 'Q' from centre 'O')

Force on point charge q: $\vec{F}_q = 0$ (using result (iii) & charge on S_1 uniform)

27. An uncharged conductor of inner radius R_1 and outer radius R_2 contains a point charge q placed at point P (not at the centre) as shown in figure? Find out the following :

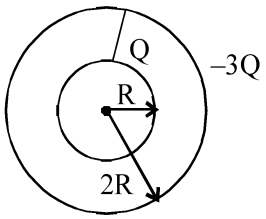


(i) V_C (ii) V_A (iii) V_B (iv) E_A
(v) E_B (vi) force on charge Q if it is placed at B

Sol. (i) $V_C = \frac{Kq}{CP} + \frac{K(-q)}{R_1} + \frac{Kq}{R_2}$ (ii) $V_A = \frac{Kq}{R_2}$ (iii) $V_B = \frac{Kq}{CB}$

(iv) $E_A = O$ (point is inside metallic conductor) (v) $E_B = \frac{Kq}{CB^2}\hat{CB}$ (vi) $F_Q = \frac{KQq}{CB^2}\hat{CB}$

28. The two conducting spherical shells are joined by a conducting wire and cut after some time when charge stops flowing.Find out the charge on each sphere after that.

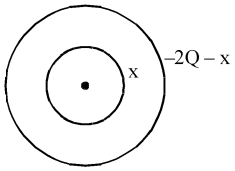


Sol. After cutting the wire, the potential of both the shells is equal

Thus, potential of inner shell $V_{in} = \frac{Kx}{R} + \frac{K(-2Q-x)}{2R} = \frac{k(x-2Q)}{2R}$

and potential of outer shell

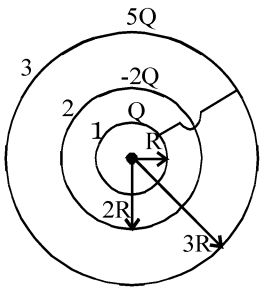
$V_{out} = \frac{Kx}{2R} + \frac{K(-2Q-x)}{2R} = \frac{-KQ}{R}$



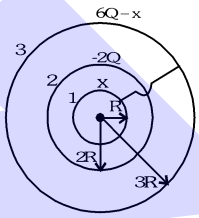
As $V_{out} = V_{in}$ $\Rightarrow \frac{-KR}{R} = \frac{K(x-2Q)}{2R} \Rightarrow -2Q = x - 2Q$ $\Rightarrow x = 0$

So charge on inner spherical shell = 0 and outer spherical shell = – 2Q.

29. Find charge on each spherical shell after joining the inner most shell and outer most shell by aconducting wire. Also find charges on each surface.



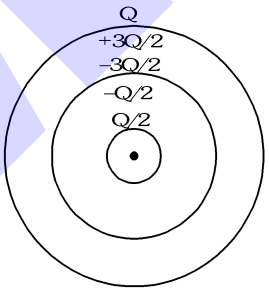
Sol. Let the charge on the innermost sphere be x.



Finally potential of shell 1 = Potential of shell 3

$$\frac{Kx}{R}+\frac{K(-2Q)}{2R}+\frac{K(6Q-x)}{3R}=\frac{KQ}{3R}+\frac{k(-2Q)}{3R}+\frac{k(5Q)}{3R}$$

$3x - 3Q + 6Q - x = 4Q$; $2x = Q$; $x = \frac{Q}{2}$



Charge on innermost shell = $\frac{Q}{2}$, charge on outermost shell = $\frac{5Q}{2}$

middle shell = –2Q

Final charge distribution is as shown in figure.

30. An isolated conducting sphere of charge Q and radius R is connected to a similar uncharged sphere (kept at a large distance) by using a high resistance wire. After a long time what is the amount of heat loss ?

Sol. When two conducting spheres of equal radius are connected charge is equally distributed on them.

So we can say that heat loss of system

$$\Delta H = U_i - U_f = \left(\frac{Q^2}{8\pi\varepsilon_0 R}-0\right)-\left(\frac{Q^2/4}{8\pi\varepsilon_0 R}+\frac{Q^2/4}{8\pi\varepsilon_0 R}\right)=\frac{Q^2}{16\pi\varepsilon_0 R}$$